Introduction to Stratospheric Dynamics

1D, 2D, Zonal mean 3D

Thermal structure of the atmosphere:

- Troposphere:
  - (They can answer this one!)

- Stratosphere:
  - Absorption of solar UV by O3
  - Emission of IR by CO2

1D/First order guess?

Radiative equilibrium
Thermal wind.

Consider a zonally symmetric atmosphere, statistical equilibrium.

Heat budget:
\[ \nu \frac{\partial T}{\partial y} + w \left( \frac{\partial T}{\partial z} + \frac{g}{c_p} \right) = Q \]

Poles, Equator \( \frac{\partial T}{\partial y} = 0 \), so \( w \left( \frac{\partial T}{\partial z} + \frac{g}{c_p} \right) = Q \)

'(Draw in vertical arrows now)

When \( Q < 0 \), the atmosphere is warmer than \( T_e \)

Continuity:
\[ \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial (\rho w)}{\partial z} \Rightarrow \frac{[V]}{L} = \frac{[w]}{H} \]

(Draw in horizontal arrows)

Magnitude of Newtonian cooling \( Q = -\alpha (T - T_e) \), \( \alpha \sim 30 \text{ days}^{-1} \)
Angular momentum considerations – cannot just flow across contours of angular momentum. Need easterly angular momentum in winter stratosphere and mesosphere, westerly in summer.

What can deposit such momentum where needed?

Eddies.

**Conventional mean framework**

Zonal mean momentum equation, zonal, log \( p \)

\[
\frac{\partial u}{\partial t} + \nabla \frac{\partial}{\partial y} \bar{u} + \bar{w} \frac{\partial}{\partial z} u - \frac{f}{\rho} \nabla \cdot (\rho \bar{u} \bar{u}')
\]

Advection by eddy component of meridional velocity of the eddy component of zonal momentum

\( \rho \bar{u} \bar{u}' \) "eddy" flux of momentum

Zonal mean in log \( p \) and deviations from this zonal mean. "Eddy" is relative.

**Isentropic mean framework**

What I advocate for, but not commonly used

- models output on pressure levels
- not traditionally used, so people have trouble interpreting.
Transformed Eulerian Mean Framework

A compromise — pressure coordinates, eddy component in \( \theta \) isolated.

QG, \( \beta \) plane, isentropic slopes are small, such that vertical motion is small

\( \Theta_z \) is to leading order only a function of \( z \). (Static stability)

Leading order, geostrophy

\( u = -\Psi_y \); \( v = \Psi_x \); \( w = 0 \)

\( \Psi \) is the geostrophic streamfunction.

QG

1. \( (\partial_t + u \partial_x + v \partial_y) u - \beta v v - f_0 v u = G(x) \)
2. \( (\partial_t + u \partial_x + v \partial_y) v + \beta y u + f_0 v u = G(y) \)

Thermodynamic equation

3. \( (\partial_t + u \partial_x + v \partial_y) \theta + w_\alpha \theta_0 z = \alpha_\theta \frac{J}{\rho} \left( \frac{\theta}{\hbar (\gamma T + g z)} \right)^{-1} \)

Combine to get approx eq:

\[
(\partial_t + u \partial_x + v \partial_y) q = X,
\]

\( q = f_0 + \beta y + \Delta^2 \Psi; \quad \Delta^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{1}{\rho_0} \frac{\partial^2}{\partial z^2} \left( \frac{\rho v^2}{N^2} \right) \)

\[
X = G(x) - G(u) + \frac{f_0}{\rho} \left[ \frac{\partial}{\partial \gamma} (\gamma T + g z) \right] \frac{J}{\alpha_\theta \hbar (\gamma T + g z)^{-1}}
\]
Zonal mean of (1)

\[ \overline{U_t} - f_0 \overline{V_a} = \frac{G\theta}{\nu} - (\overline{U'V'})_y \]

\[ \overline{V} = \overline{V_x} = 0 \]

Zonal mean of (3)

\[ \frac{\partial \overline{\theta}}{\partial t} + \overline{u_a} \theta_z = \frac{1}{\rho} \left[ \frac{2}{\partial (\rho_T + \rho \theta)} \right]^{\perp} - \nabla^T \theta_y \]

continuity \( \overline{v_a} \theta_y + \frac{1}{\rho} (\rho \overline{u_a})_z = 0 \)

\( f_0 \overline{u}_z = - \frac{K}{H} \pi \overline{\theta}_y \)

or just

\[ f_0 \overline{u}_z \approx - \overline{\theta}_y \]

Continuity lets us define

\[ (\overline{v_a}, \overline{w_a}) = \left[ \frac{1}{\rho} \frac{\partial (\rho X_a)}{\partial z}, - \frac{\partial X_a}{\partial y} \right] \] an ageostrophic streamfunction.

New streamfunction also nondivergent

\[ (\overline{\nu_*}, \overline{w_*}) = \left[ \frac{1}{\rho} \frac{\partial (\rho X_*)}{\partial z}, - \frac{\partial X_*}{\partial y} \right] \]

\[ X_* = X_a + X_c \]
Sub \( \chi_k \) into (4)

\[
\frac{\partial \overline{\Theta}}{\partial t} + \overline{w} \cdot \overline{\Theta}_0 z = \overline{\mathcal{J}} - \frac{2}{\rho} \frac{\partial}{\partial y} \left( \overline{v' \Theta'} \right) - \overline{\Theta}_0 z \frac{\partial \chi_c}{\partial y}
\]

Choose \( \chi_c = -\frac{\overline{v' \Theta'}}{\overline{\Theta}_0 z} \),

\( \chi_k = \chi_a - \frac{\overline{v' \Theta'}}{\overline{\Theta}_0 z} \) residual streamfunction.

\[
\frac{\partial \overline{\Theta}}{\partial t} + \overline{w} \cdot \overline{\Theta}_0 z = \frac{\overline{\mathcal{J}}}{\rho \Pi}, \text{ and if we make the same substitution into momentum equation,}
\]

\[
\frac{\partial \overline{u}}{\partial t} - f_0 \overline{\chi}_k = \overline{G}_x + \frac{1}{\rho} \nabla \cdot F
\]

\[
F = \begin{pmatrix} \overline{F}(y) \\ \overline{F}(z) \end{pmatrix} = \begin{pmatrix} -\rho \overline{u v'} \\ \rho f_0 \overline{v' \Theta'}/\overline{\Theta}_0 z \end{pmatrix} \text{ is the Eliassen-Palm flux}
\]

【thermals wind is the same, continuity the usual, by our \( \chi_k \) definition】

\( \frac{1}{\rho} \nabla \cdot F \) entirely summarizes the eddy forcing on the mean state.

residual circulation corresponds to density-weighted mean circulation in isentropic coordinates.
Upward propagating waves, planetary scale wave numbers 1, 2, 3 reach a point where they cannot continue to propagate and break. (Depends on characteristics of mean flow)

\[ \tilde{\omega}^* \text{ from continuity} \]
\[ \frac{\partial}{\partial z} (\rho \tilde{\omega}^*) = \frac{1}{f_0} \frac{\partial}{\partial y} (\nabla \cdot F) \]

\[ \rho \tilde{\omega}^* \rightarrow 0 \text{ as } z \rightarrow \infty \]

So if upward propagating wave, \( F_y = 0 \),
\[ \nabla \cdot F = \frac{\partial F_z}{\partial z} \]

\( F_z \) must vanish at large \( z \).

Integrate down from \( z \rightarrow \infty \)
\[ \rho \tilde{\omega}^* = \frac{1}{f_0} \frac{\partial F_z}{\partial y} \]

\( \tilde{\omega}^* \) only cares about what happens above.